Modified entropic gravity revisited

Tower Wang[*1]

[1]*Department of Physics, East China Normal University,*
Shanghai 200241, China

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Abstract

Inspired by Verlinde's idea, some modified versions of entropic gravity have appeared in the literature. Extending them in a unified formalism, we derive the generalized gravitational equations accordingly. From gravitational equations, the energy-momentum conservation law and cosmological equations are investigated. The covariant conservation law of energy-momentum tensor severely constrains viable modifications of entropic gravity. A discrepancy arises when two independent methods are applied to the homogeneous isotropic universe, posing a serious challenge to modified models of entropic gravity.

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[*] Electronic address: twang@phy.ecnu.edu.cn

I.   INTRODUCTION

In the past forty years, our understanding about the nature of gravity has been much enriched by the thermodynamics of gravity. The black hole thermodynamics was well established, and the holographic principle was rigorously realized in AdS/CFT correspondence. Encouraged by these achievements, it was conjectured that gravity is an emergent phenomenon. In other words, gravity may not be a fundamental force.

An intriguing way towards emergent gravity is making Einstein's equations from the Clausius relation between entropy change and heat flux [1], or more recently, from the equipartition relation between energy and bits [2]. In the latter scenario, gravity is deemed as an entropic force, and the derivation of gravitational equations [2] relies on three ingredients: the Unruh law [3] identifying temperature with the local acceleration; the holographic principle ensuring that the number of bits is proportional to the area of holographic screen; and an equipartition rule [4] relating energy or mass to the temperature and the number of bits.

Soon after the proposal, entropic gravity was widely studied.[1] Interestingly, a lot of modified models of entropic gravity emerged. Almost all of them can be classified broadly into three categories, corresponding to modifying one of the ingredients above. First, applying a different relation between temperature and acceleration, modified Newtonian dynamics was reinterpreted in [13–15] through the entropic force. Second, deformed entropy-area relations motivated further investigations on modified gravity in [16–26]. Third, Refs. [27–35] explored the possibility of entropic gravity with more complicated equipartition relations. Exceptions include changing more than one ingredients [36] or considering corrections from the uncertainty principle [37–39].

In Verlinde's enrtopic gravity scenario, the Unruh law $k_B T = \hbar a/(2\pi c)$, the holographic relation $N = Ac^3/(G\hbar)$ and the equipartition rule $E = Nk_B T/2$ are combined to give

$$E = \frac{aAc^2}{4\pi G}.$$

(1)

Taking E as the total energy inside a closed holographic surface, and identifying A as the surface area and a the (red-shifted) surface acceleration, one is able to build up the Newton's (Einstein's) gravitational equations [2].

It is not impossible that Einstein gravity holds exactly at all energy scales, no matter how strong or weak the gravitational field is. There is also possibility that gravity does not have an entropic origin. Otherwise, if gravity is really an entropic force and gets modified in some regions, we may have to take modified entropic gravity seriously.

In the present paper, we start our adventure by observing that most modified entropic gravity models summarized above, whichever ingredient they modify, can be formally expressed by inserting a factor $f(a, A)$ into Eq. (1),

$$E = \frac{aAc^2}{4\pi G} f(a, A).$$

(2)

Here $f(a, A)$ is a model-dependent function of acceleration and area of the holographic surface. In the relativistic case, acceleration a should be red-shifted like temperature.

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[1] The readers may refer to [5–12] as a partial list and references therein.

The rest of our paper is organized as follows. Starting from relation (2), we will derive gravitational equations in Sec. II A and deal with the static weak field limit in Sec. II B. Although the gravitational equations look fine, we find in Sec. III that viable models of modified entropic gravity are constrained tightly by the covariant conservation of energy-momentum. Under the assumption of Friedmann-Lemaître-Robertson-Walker (FLRW) metric, in Sec. IV we will write down cosmological equations, which further constrain the modified entropic gravity models as viable theories to understand our Universe. In Sec. V we discuss possible implications of our results. Some useful formulae for FLRW spacetime are relegated to Appendix A. Throughout this paper, we will follow the convention of notations in [35].

II. GRAVITATIONAL EQUATIONS

A. Field equations

To get the relativistic gravitational field equations, parallel to Ref. [2] we begin with a static background which has a time-like Killing vector ξ^μ normalized as

$$\xi_\mu \xi^\mu = -e^{2\phi} \tag{3}$$

with the Newtonian potential ϕ. Introducing a vector $\mathcal{N}^\mu$ outward normal to the holographic surface $\mathcal{S}$, we can express the red-shifted acceleration as

$$a = e^\phi \mathcal{N}^\mu \nabla_\mu \phi. \tag{4}$$

Then by virtue of $E = Mc^2$ it is easy to write Eq. (2) in the integral form

$$M = \frac{1}{4\pi G} \int_{\mathcal{S}} f e^\phi \nabla\phi \cdot dA, \tag{5}$$

where $dA_\mu = |dA|\mathcal{N}_\mu$ and

$$|dA| = dx^\alpha dx^\beta \epsilon_{\kappa\lambda\alpha\beta} e^{-\phi} \xi^\kappa \mathcal{N}^\lambda. \tag{6}$$

In this subsection, we will derive the modified field equations following the convention of notations in [35]. To avoid repetition, some results will be quoted directly from [35]. Although the present subsection is clear in outline and new details, for old details of the quoted results, we refer the reader to Sec. IV A and Appendix A of [35].

Here are some essential points from Ref. [35]. In the appendix of [35], it has been demonstrated that for a general function f, e.g. $f(a, A)$, the expression of mass (5) can be rearranged as

$$M = \frac{1}{4\pi G} \int_{\mathcal{V}} [f\mathcal{R}^\rho{}_\sigma \xi^\sigma n_\rho + (\nabla_\sigma f)(\nabla^\rho \xi^\sigma) n_\rho] dV, \tag{7}$$

in which $\mathcal{V}$ is the 3-dimensional volume bounded by the holographic screen $\mathcal{S}$, n^μ is a future-directed vector normal to $\mathcal{V}$, and $\mathcal{R}_{\mu\nu}$ denotes the Ricci tensor. As also mentioned in the appendix of [35], one may recast the second term of (7) into

$$(\nabla_\sigma f)(\nabla^\rho \xi^\sigma) n_\rho = n_\rho \nabla^\rho (\xi^\sigma \nabla_\sigma f) - n_\rho \xi^\sigma \nabla^\rho \nabla_\sigma f. \tag{8}$$

Moreover, we have shown in Ref. [35] that if one can prove $\xi^\sigma \nabla_\sigma f = 0$, then gravitational equations are of the form

$$f \mathcal{R}_{\mu\nu} - \nabla_\mu \nabla_\nu f = 8\pi G \left(\mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{T} \right), \tag{9}$$

where $\mathcal{T}_{\mu\nu}$ is the energy-momentum tensor. In Appendix A of [35], it was established that $\xi^\sigma \nabla_\sigma (e^\phi \mathcal{N}^\mu \nabla_\mu \phi) = 0$, namely $\xi^\sigma \nabla_\sigma a = 0$, hence $\xi^\sigma \nabla_\sigma f(a) = 0$ and equations (9) are correct if f is a function purely of acceleration (4).

In the rest of this subsection, we will show that $\xi^\sigma \nabla_\sigma f(a, A) = 0$ and thus (9) continue to be correct when f is a function of both acceleration a and area A generally. This can be achieved by proving the equality

$$\xi^\sigma \nabla_\sigma A = 0. \tag{10}$$

To do this, we note

$$
\begin{aligned}
\xi^\sigma \nabla_\sigma A &= \xi^\sigma \nabla_\sigma \int_{\mathcal{S}} dx^\alpha dx^\beta \epsilon_{\mu\nu\alpha\beta} e^{-\phi} \xi^\mu \mathcal{N}^\nu \\
&= \int_{\mathcal{S}} dx^\alpha dx^\beta \xi^\sigma \nabla_\sigma \left(\epsilon_{\mu\nu\alpha\beta} e^{-\phi} \xi^\mu \mathcal{N}^\nu \right).
\end{aligned} \tag{11}
$$

In the second line, we have used normalization condition (3) and the fact that the holographic screen $\mathcal{S}$ corresponds to the equipotential surface [2].

To proceed, we recall that the holographic screen $\mathcal{S}$ is a 2-dimensional surface. Restricted to this surface, we can define a 2-dimensional tensor $\omega^{\alpha\beta} = \xi_\kappa \mathcal{N}_\lambda \epsilon^{\kappa\lambda\alpha\beta}$. Making use of the equality

$$\epsilon^{\kappa\lambda\alpha\beta} \epsilon_{\mu\nu\alpha\beta} = -4 \delta^\kappa_{[\mu} \delta^\lambda_{\nu]} = \frac{-4}{2!} (\delta^\kappa_{\ \mu} \delta^\lambda_{\ \nu} - \delta^\kappa_{\ \nu} \delta^\lambda_{\ \mu}), \tag{12}$$

it is not hard to see

$$\omega^{\alpha\beta} \xi^\sigma \nabla_\sigma \left(\epsilon_{\mu\nu\alpha\beta} e^{-\phi} \xi^\mu \mathcal{N}^\nu \right) = e^{-\phi} \xi^\mu \mathcal{N}^\nu \xi_\kappa N_\lambda \epsilon^{\kappa\lambda\alpha\beta} \xi^\sigma \nabla_\sigma \epsilon_{\mu\nu\alpha\beta} - 2(\xi_\mu N_\nu - \xi_\nu N_\mu) \xi^\sigma \nabla_\sigma \left(e^{-\phi} \xi^\mu \mathcal{N}^\nu \right). \tag{13}$$

It is interesting to manipulate indices of the first term of (13) as

$$
\begin{aligned}
e^{-\phi} \xi^\mu \mathcal{N}^\nu \xi_\kappa N_\lambda \epsilon^{\kappa\lambda\alpha\beta} \xi^\sigma \nabla_\sigma \epsilon_{\mu\nu\alpha\beta} &= e^{-\phi} \xi_\mu \mathcal{N}_\nu \xi^\kappa N^\lambda \epsilon_{\kappa\lambda\alpha\beta} \xi^\sigma \nabla_\sigma \epsilon^{\mu\nu\alpha\beta} \\
&= e^{-\phi} \xi_\kappa \mathcal{N}_\lambda \xi^\mu N^\nu \epsilon_{\mu\nu\alpha\beta} \xi^\sigma \nabla_\sigma \epsilon^{\kappa\lambda\alpha\beta} \\
&= e^{-\phi} \xi^\mu \mathcal{N}^\nu \xi_\kappa N_\lambda \xi^\sigma \nabla_\sigma \left(\frac{1}{2} \epsilon^{\kappa\lambda\alpha\beta} \epsilon_{\mu\nu\alpha\beta} \right) \\
&= 0.
\end{aligned} \tag{14}
$$

The last line follows from (12) and $\nabla_\sigma \delta^\mu_{\ \nu} = 0$. In Ref. [35] it has been shown that

$$\xi^\mu \nabla_\mu \phi = -e^{-2\phi} \xi^\mu \xi^\nu \nabla_\mu \xi_\nu = 0, \quad \mathcal{N}_\nu \nabla^\mu \mathcal{N}^\nu = 0. \tag{15}$$

Therefore the second term of (13) is simplified,

$$-2(\xi_\mu N_\nu - \xi_\nu N_\mu) \xi^\sigma \nabla_\sigma \left(e^{-\phi} \xi^\mu \mathcal{N}^\nu \right) = 2e^{-\phi} \xi_\nu N_\mu \xi^\sigma \nabla_\sigma \left(\xi^\mu \mathcal{N}^\nu \right) = 0. \tag{16}$$

Here the orthogonal relation $\mathcal{N}^\mu \xi_\mu = 0$ is taken into account. Putting Eqs. (13), (14) and (16) together, it is apparent that

$$\omega^{\alpha\beta} \xi^\sigma \nabla_\sigma \left(\epsilon_{\mu\nu\alpha\beta} e^{-\phi} \xi^\mu \mathcal{N}^\nu \right) = 0. \tag{17}$$

At the same time, restricted to the holographic screen $\mathcal{S}$, all 2-dimensional anti-symmetric covariant tensor should be proportional to $\omega_{\alpha\beta}$ and thus proportional to each other in the same basis. That is to say,

$$\xi^\sigma \nabla_\sigma \left(\epsilon_{\mu\nu\alpha\beta} e^{-\phi} \xi^\mu \mathcal{N}^\nu \right)\big|_\mathcal{S} = 0 \tag{18}$$

and subsequently

$$\int_\mathcal{S} dx^\alpha dx^\beta \xi^\sigma \nabla_\sigma \left(\epsilon_{\mu\nu\alpha\beta} e^{-\phi} \xi^\mu \mathcal{N}^\nu \right) = 0. \tag{19}$$

This conclude our proof of Eq. (10).

Since we have demonstrated $\xi^\sigma \nabla_\sigma A = 0$ in the above, and Ref. [35] has verified that $\xi^\sigma \nabla_\sigma a = 0$, it is straightforward to obtain

$$\xi^\sigma \nabla_\sigma f(a, A) = f_{,a} \xi^\sigma \nabla_\sigma a + f_{,A} \xi^\sigma \nabla_\sigma A = 0 \tag{20}$$

as promised earlier in this section. We can conclude that, for modified entropic gravity models of form (2), when f is a general function of acceleration a and area A, the gravitational field equations are given by (9). In Verlinde's entropic gravity model, $f = 1$, and Eqs. (9) reduce to the Einstein equations.

B. Static weak field limit

The static weak field limit provides not only a crosscheck of our calculations in Sec. II A, but also a playground to test modified models of entropic gravity. Hence it would be valuable to take a closer look at this limit.

First, we note that the Newtonian limit corresponds to the 00-component of relativistic gravitational equations. For this component, the second term of the left hand side of (9) reads

$$-\nabla_0 \nabla_0 f = -\partial_0 \partial_0 f + \Gamma_{00}^\mu \partial_\mu f. \tag{21}$$

In the static case, both temperature and entropy of the holographic surface is time-independent, hence $\partial_0 \partial_0 f$ vanishes. The weak gravitational field $g_{\mu\nu}$ can be expanded near the Minkowski metric $\eta_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1. \tag{22}$$

For a static metric, we have $\partial_0 g_{\mu\nu} = 0$ and

$$\Gamma_{00}^\mu = -\frac{1}{2} \eta^{\mu\lambda} \partial_\lambda h_{00}, \quad \mathcal{R}_{00} = -\frac{1}{2} \nabla^\lambda \nabla_\lambda h_{00} \tag{23}$$

in which $h_{00} \simeq -2\phi$. Therefore the 00-component of (9) on the left hand side becomes

$$\begin{aligned} f \mathcal{R}_{00} - \nabla_0 \nabla_0 f &\simeq f \eta^{\mu\lambda} \partial_\mu \partial_\lambda \phi + \eta^{\mu\lambda} (\partial_\lambda \phi) \partial_\mu f \\ &= \eta^{\mu\lambda} \partial_\mu (f \partial_\lambda \phi). \end{aligned} \tag{24}$$

On the other hand, in the Newtonian limit, the stress-energy tensor $\mathcal{T}_{\mu\nu} = \rho u_\mu u_\nu$ with $u_\mu = (\sqrt{-g_{00}}, 0, 0, 0)$. As a result, we find in this limit the Poisson's equation is generalized to

$$\delta^{ij} \partial_i (f \partial_j \phi) \simeq 4\pi G \rho \tag{25}$$

5

where indices i, j represent spatial coordinates. In the case $f = 1$, this equation reduces to the Newton's law of gravity.

The above generalized Poisson's equation can be also obtained directly by writing (5) as

$$\int_{\mathcal{V}} \rho dV \simeq \frac{1}{4\pi G} \int_{\mathcal{S}} \delta^{ij} f \partial_j \phi \cdot dA_i \tag{26}$$

in the static weak field limit and applying the divergence theorem. This confirms that our calculation is consistent.

A brief comment is in order here. If function f deviates obviously from constant or from 1, with the help of Eq. (25), we can test modified entropic gravity models against observations such as planetary orbits.

III. ENERGY-MOMENTUM CONSERVATION LAW

The conservation of energy is a fundamental law in physics. In general relativity, it has a nice generalization: the covariant conservation of energy-momentum tensor, formulated as $\nabla_\nu \mathcal{T}_\mu{}^\nu = 0$ and guaranteed by the fact that the covariant divergence of Einstein tensor is zero,

$$\nabla_\nu \left(\mathcal{R}_\mu{}^\nu - \frac{1}{2} \delta_\mu{}^\nu \mathcal{R} \right) = 0. \tag{27}$$

In modified entropic gravity, gravitational field equations (9) can be transformed to

$$f \left(\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} \right) - \left(\nabla_\mu \nabla_\nu f - \frac{1}{2} g_{\mu\nu} \nabla^2 f \right) = 8\pi G \mathcal{T}_{\mu\nu}, \tag{28}$$

more complicated than the counterparts in Einstein gravity, so in this new situation we have to reconsider the law of energy-momentum conservation.

For this purpose, we will work out the covariant derivative of the left hand side of Eq. (28). Remind that for a vector $\nabla^\nu f$, one has

$$\nabla_\nu \nabla_\mu \nabla^\nu f - \nabla_\mu \nabla_\nu \nabla^\nu f = \mathcal{R}_{\mu\lambda} \nabla^\lambda f \tag{29}$$

by definition of the Riemann tensor. Thanks to this identity, after a little algebra, one may directly demonstrate

$$8\pi G \nabla_\nu \mathcal{T}_\mu{}^\nu = -\frac{1}{2} \left(\mathcal{R} \nabla_\mu f + \nabla_\mu \nabla^2 f \right). \tag{30}$$

That is, the energy-momentum tensor cannot be covariantly conserved in modified entropic gravity unless

$$\mathcal{R} \nabla_\mu f + \nabla_\mu \nabla^2 f = 0. \tag{31}$$

Only a narrow subset of models are permitted by this requirement.

The situation is reminiscent of the so-called Chern-Simons modified general relativity [40], in which the energy-momentum conservation is unwarranted by gravitational equations. In the literature, this is not taken as a fatal defect of the Chern-Simons modified gravity, though one should be careful with it when seeking for exact metric solutions. Likewise, we can say Eq. (31) is a criterion of admissible metrics in modified models of entropic gravity.

IV.   COSMOLOGICAL EQUATIONS

In Refs. [41, 42], Friedmann equations are obtained from Verlinde's model [2]. In modified models (2) of entropic gravity, it is possible to derive cosmological equations along the same line. To avoid confusions in notation, we use $R(t)$ to denote the scale factor. Another method to write down cosmological equations is by directly applying gravitational equations (9) to cosmological background. We will explore both approaches, assuming the FLRW metric

$$ds^2 = -dt^2 + R^2 \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right]. \tag{32}$$

Let us consider Eq. (2) on a sphere with physical radius Rr in the FLRW universe. On the one hand, the red-shifted acceleration and area are now expressed as $a = -\ddot{R}r$ and $A = 4\pi R^2 r^2$. On the other hand, $E = Mc^2$, and the active gravitational mass [43] is given by

$$M = \int_{\mathcal{V}} (\rho + 3p) dV = \int_{\mathcal{V}} (\rho + 3p) 4\pi R^3 r^2 dr \tag{33}$$

for a perfect fluid whose stress-energy tensor $\mathcal{T}_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$ with $u_\mu u^\mu = -1$. In accordance with metric (32), it is convenient to choose $u_\mu = (1, 0, 0, 0)$. Then Eq. (2) leads to the acceleration equation

$$\frac{\ddot{R}}{R} f = -\frac{4\pi G}{r^3} \int_{\mathcal{V}} (\rho + 3p) r^2 dr. \tag{34}$$

This equation combined with continuity equation

$$\dot{\rho} + \frac{3\dot{R}}{R}(\rho + p) = 0 \tag{35}$$

dictates the background dynamics of cosmology. Especially, putting them together, we find

$$\dot{R}\ddot{R} f = \frac{4\pi G}{r^3} \frac{d}{dt} \int_{\mathcal{V}} \rho R^2 r^2 dr. \tag{36}$$

It is useful to recall that continuity condition (35) comes from the covariant conservation law of energy-momentum. Besides this condition, energy-momentum conservation $\nabla_\nu \mathcal{T}_\mu{}^\nu = 0$ indicates also $\partial_r p = \partial_\theta p = \partial_\varphi p = 0$. With these conditions imposed, Eqs. (34) and (36) are integrated as

$$\frac{4\pi G}{r^3} \int_{\mathcal{V}} \rho R^2 r^2 dr = -R\ddot{R}f - 4\pi G p R^2 = \int \dot{R}\ddot{R}f dt, \tag{37}$$

yielding a differential equation

$$\partial_t(R^2 \ddot{R} \partial_r f) = 0 \tag{38}$$

which is apparently solved by

$$f = \frac{g_1(r)}{R^2 \ddot{R}} + g_2(t). \tag{39}$$

Remember that f is a function of a and A, so the expression of f should have the form

$$f = \frac{C_1}{aA} + g_2\left(aA^{-1/2}\right). \tag{40}$$

Here C_1 is a constant, while g_2 is a function of $\ddot{R}R^{-1}$.

As implied by Eq. (34), if $C_1 \neq 0$, then density ρ will be dependent of radial coordinate r, violating the Copernican principle. Therefore, to get rid of exotic cosmology with radial dependence, one may set $C_1 = 0$ and treat f as a function of $aA^{-1/2}$. It is remarkable that the cosmologically viable modified entropic gravity is restricted to such a small subset of models, simply by covariant conservation of energy momentum and the Copernican principle. We will investigate this subset of cosmologically viable models elsewhere.

Since we have established gravitational equations (9) in Sec. II A, there is another approach to cosmological equations. That is applying Eqs. (9) to the FLRW metric (32). By doing this, we find

$$-\frac{3\ddot{R}}{R}f - \partial_t^2 f = 4\pi G(\rho + 3p),$$

$$\partial_t \partial_r f - \frac{\dot{R}}{R}\partial_r f = 0,$$

$$\left(\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2}\right)f + \frac{\dot{R}}{R}\partial_t f - \frac{1}{R^2}\partial_r^2 f = 4\pi G(\rho - p),$$

$$\left(\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2}\right)f + \frac{\dot{R}}{R}\partial_t f - \frac{1}{R^2 r}\partial_r f = 4\pi G(\rho - p). \tag{41}$$

These gravitational equations put very stringent limits on f by differential equations

$$\partial_r \left(\frac{1}{r}\partial_r f\right) = \partial_t \left(\frac{1}{R}\partial_r f\right) = 0, \tag{42}$$

whose solution is

$$f = C_1 R r^2 + g_2(t). \tag{43}$$

Because f is a function of a and A, the admissible expression should be of the form

$$f = g_2\left(aA^{-1/2}\right). \tag{44}$$

Here g_2 is a function of $\ddot{R}R^{-1}$ again but the constant $C_1 = 0$. Intriguingly, the Copernican principle is automatically satisfied.

Unfortunately, it is very difficult to reproduce acceleration equation (34) from gravitational equations (41). From (41) we obtain

$$-\frac{3\ddot{R}}{R}f - \partial_t^2 f + \frac{\ddot{R}r}{\dot{R}}\partial_t \partial_r f - \frac{\ddot{R}r}{R}\partial_r f = 4\pi G(\rho + 3p). \tag{45}$$

This equation can be integrated to give (34) if redundant terms vanish,

$$-\partial_t^2 f + \frac{\ddot{R}r}{\dot{R}}\partial_t \partial_r f = 0. \tag{46}$$

For a general function of f, the condition is not always satisfied and hence we cannot recover Eq. (34). This discrepancy is not attributed to any error in our calculation. It stems from a technical trick in Sec. II A: we began with a static background which has a time-like

Killing vector [2]. For the FLRW spacetime, this is possible only if metric (32) reduces to the de Sitter or Minkowski spacetime [44]. Indeed, it is checkable that for a general function $f(a, A)$, condition (46) is assured if $R = \exp(Ht)$ with a constant H. In other words, Eq. (34) and Eqs. (41) are consistent in static cases, though the inconsistency persists in other cases.

The issue discussed above present a new challenge to modified models of entropic gravity. So far we do not have a perfect solution to this challenge. At this point we mention several possibilities. First, possibly the FLRW metric is not an exact solution of modified entropic gravity models but needs adjustment. The second possible attitude is insisting on (34) and taking (46) as a consistency condition for allowable models. Note that this condition is met by Einstein gravity ($f = 1$). The third possible way to alleviate the contradiction is tuning the definition of red-shifted "temperature" for the FLRW universe in modified entropic gravity,

$$T \propto -\ddot{R}r + \frac{R}{\dot{R}r^2 f} \int_{\mathcal{V}} \left[\partial_t(\dot{R}r)\partial_r\partial_t f - \partial_r(\dot{R}r)\partial_t^2 f \right] r^2 dr, \tag{47}$$

giving rise to a correction term to (34). The fourth but very difficult solution is deriving gravitational equations without assuming a static background, then probably we will get field equations different from (9). Before arriving at the final answer, it is still too early to claim which possibility is more promising or whether there are other possibilities.

In accordance with gravitational equations (9), we can also study condition (31) of covariant energy-momentum conservation. Assuming the FLRW metric (32), this condition becomes

$$6 \left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} \right) \nabla_\mu f + \nabla_\mu \nabla^2 f = 0, \tag{48}$$

or explicitly

$$\left(\frac{5\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} \right) \partial_r f = 0,$$

$$\left(\frac{3\ddot{R}}{R} + \frac{9\dot{R}^2}{R^2} \right) \partial_t f - \partial_t^3 f - \frac{3\dot{R}}{R}\partial_t^2 f - \frac{3\dot{R}}{R^3 r}\partial_r f = 0. \tag{49}$$

Putting (44) into these equations, we are led to a severe constraint. The constraint rules out nearly all models of form (2) other than Einstein gravity.

V. DISCUSSION

In this paper, we unified a number of modified entropic gravity models in the literature to a general form (2). The corresponding gravitational equations are demonstrated to be (9), whose static weak field limit is consistent with a straightforward calculation. If the modified model deviates significantly from Einstein gravity or Newtonian gravity, the static weak field equation (25) provides an arena to test them against observations.

In these models, the energy-momentum tensor is covariantly conserved if and only if condition (31) is met. The condition can be regarded as a constraint on viable models, or reverse the logic, a constraint on suitable metric solutions for modified entropic gravity.

Assuming the FLRW metric, we derived cosmological equations in two independent approaches. To our surprise, a discrepancy exists unless condition (46) is satisfied. There are several possible ways to reconcile this discrepancy. One way is taking (46) as a consistency condition of admissible models, then Einstein gravity ($f = 1$) wins out.

As indicated by our results, the modified entropic gravity models of form (2), if not killed, should live in a very narrow room to assure the energy-momentum conservation and to accommodate a homogeneous isotropic universe.

Acknowledgments

Appendix A: Useful formulae

In this appendix, we gather some formulae for spatially flat FLRW spacetime (32). These formulae are useful in Sec. IV.

In our notations, the nonvanishing Christoffel connections are

$$\Gamma^t_{rr} = R\dot{R}, \quad \Gamma^t_{\theta\theta} = R\dot{R}r^2, \quad \Gamma^t_{\varphi\varphi} = R\dot{R}r^2\sin^2\theta,$$
$$\Gamma^r_{\theta\theta} = -r, \quad \Gamma^r_{\varphi\varphi} = -r\sin^2\theta, \quad \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta,$$
$$\Gamma^r_{tr} = \Gamma^\theta_{t\theta} = \Gamma^\varphi_{t\varphi} = \frac{\dot{R}}{R}, \quad \Gamma^\theta_{r\theta} = \Gamma^\varphi_{r\varphi} = \frac{1}{r}, \quad \Gamma^\varphi_{\theta\varphi} = \cot\theta, \tag{A1}$$

which lead to the nonzero components of the Ricci tensor

$$\mathcal{R}_{tt} = -\frac{3\ddot{R}}{R}, \quad \mathcal{R}_{rr} = R\ddot{R} + 2\dot{R}^2,$$
$$\mathcal{R}_{\theta\theta} = \left(R\ddot{R} + 2\dot{R}^2\right)r^2, \quad \mathcal{R}_{\varphi\varphi} = \left(R\ddot{R} + 2\dot{R}^2\right)r^2\sin^2\theta \tag{A2}$$

and the Ricci scalar

$$\mathcal{R} = 6\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right). \tag{A3}$$

When $H = \dot{R}/R$ is restricted to be constant, the FLRW metric reduces to the de Sitter spacetime. This spacetime has a timelike Killing vector

$$\xi^\mu\partial_\mu = \partial_t - Hr\partial_r, \tag{A4}$$

giving rise to $e^{2\phi} = 1 - H^2R^2r^2$. The other useful formulae for the de Sitter spacetime are

$$\mathcal{N}^\mu\partial_\mu = e^{-\phi}\left(-HRr\partial_t + \frac{1}{R}\partial_r\right),$$
$$u^\mu\partial_\mu = n^\mu\partial_\mu = \partial_t. \tag{A5}$$

It is trivial to check $a = e^\phi\mathcal{N}^\mu\nabla_\mu\phi = -H^2Rr$.

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